MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Ocean Engineering

Cambridge, Massachusetts 02139

ADA039912

AD No.
DDC FILE COPY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF OCEAN ENGINEERING .

CAMBRIDGE, MASS. 02139

$See \ 1423$

IMPULSIVE LOADING OF AN IDEAL
FIBRE-REINFORCED RIGID-PLASTIC BEAM

by

F. Laudiero[†]

and

Norman Jones

Report 77 - 3

March 1977

Distribution Unlimited

[†] On leave from University of Bologna, Facoltà di Ingegneria,
Bologna, Italy

Table of Contents

Summary

This article examines the dynamic plastic response of an
impulsively loaded ideal fibre-reinforced (strongly anisotropic)
rigid-plastic beam. It is demonstrated that the response duration
and permanent transverse displacements are significantly less than
the corresponding quantities in an "equivalent" rigid perfectly
plastic isotropic beam.

Notation

a	location of plastic hinge (Figure 2(b))
c	$(Q_1/m)^{1/2}$
m	mass per unit length
n	strain hardening index in equation (1)
q	Q_1/Q_o
t	time
t_f	response duration
t_{fi}	response duration for a rigid perfectly plastic isotropic beam
w	displacement in z sense
w_f	permanent displacement in -z sense
w_{fi}	maximum permanent displacement in a rigid perfectly plastic isotropic beam
$x,y,z,$	cartesian coordinates defined in Figure 1
H	beam thickness
$2L$	beam span
M_o	$\sigma_o H^2 / 4$
Q	transverse shear force defined in Figure 1
Q_o, Q_1	parameters defined by equation (1)
Q_p	transverse shear force defined by equation (1)
V_o	impulsive velocity
β	λq
γ	shear strain
λ	V_o/c
σ_o	uniaxial yield stress
$(\dot{\ })$	$\frac{\partial}{\partial t}(\ )$.

Introduction

The dynamic plastic response of some ideal fibre-reinforced (strongly anisotropic) rigid-plastic beams were examined recently in References [1] to [6] using a continuum theory which made no distinction between the behavior of the fibres and the response of the matrix. These theoretical predictions were particularly attractive because the main features of response were obtained with little effort. Moreover, it was demonstrated in Reference [2] that the response durations and permanent transverse displacements of several ideal fibre-reinforced beams loaded dynamically were less than the corresponding values for similar rigid-plastic isotropic beams.

The beams examined in all the previous References ([1]-[6]) were impacted transversely by a point mass $M^{\dagger}$ travelling with an initial velocity V_o. It is the purpose of this present note to seek the dynamic plastic response of a beam which is subjected to an impulsive transverse velocity V_o distributed uniformly over the entire span. The particular case of a clamped beam is studied herein because the experimental arrangement required for tests is fairly straightforward and comparisons may be made with the corresponding theoretical and experimental results which have been reported in the literature for the dynamic plastic response of similar isotropic beams [7].

$\dagger$ The point mass M is infinite in References [1] and [3] as discussed in Reference [2].

Basic Equations

The theoretical foundations on which the analysis in this note rests are described in References [1] and [2] to which a reader is referred for further details. The material is assumed to be incompressible and inextensible in the direction of the fibres along a beam axis. In addition, the deformations are assumed to remain infinitesimal, while material elasticity and transverse wave propagation effects are neglected: simplifications which are customarily made in theoretical work on the dynamic plastic response of isotropic beams [8-10]. These assumptions imply that only shear deformations can develop in ideal fibre-reinforced beams.

The material is assumed to be rigid strain hardening [3].

$$Q_p = \pm \{Q_o + Q_1 |\gamma|^n\} \qquad (1)^\dagger$$

where $\gamma = \partial w/\partial x$ is the shear strain and Q_o, Q_1 and n are positive constants. Plastic flow of the material is controlled by the magnitude of the transverse shear force Q_p as discussed in Reference [2], and the material remains rigid during unloading.

The equation of transverse equilibrium for a beam is

$$\partial Q/\partial x + P(x,t) = m\partial^2 w/\partial t^2 \qquad (2)$$

where the various symbols are defined in Figure 1 and in the accompanying notation. If an element of a beam at $x = a$ is crossed by a velocity discontinuity which travels with a velocity $\dot{a}$ then the conservation of momentum (dynamic jump condition) demands

$$[Q(a, t)] = - m\dot{a}[\partial w/\partial t] \qquad (3)$$

---

† The theoretical analyses in References [1] and [2] were developed for a rigid linear strain hardening material with $n = 1$.

at x = a, while continuity of displacement (kinematic jump condition) requires

$$\dot{a}[\partial w/\partial x] = - [\partial w/\partial t] \qquad (4)$$

at x = a, where [X] means $X(a^+) - X(a^-)$ and denotes the difference in X on either side of the discontinuity at x = a(t). In passing, it should be remarked that equations (3) and (4) are identical to equations (19a) and (21a) of Symonds [11].

Clamped Beam Loaded Impulsively

Consider a stationary ideal fibre-reinforced rigid-plastic beam which is subjected to time t = 0 to a velocity V_o distributed uniformly over the entire span 2L as shown in Figure 2(a). Large transverse shear forces (Q_p) will develop immediately at the fully clamped supports and give rise to a slope discontinuity (hereafter called a "shear hinge") according to equation (1). These shear hinges propagate with a velocity $\dot{a}$ inwards from both supports towards the beam centre as indicated in Figure 2(b). Plastic deformation is confined to the two travelling shear hinges, while the central portion of the beam $-a \le x \le a$ moves as a rigid body with a velocity V(t) and the two outer regions $a \le x \le L$ and $-L \le x \le -a$ remain stationary. This problem is symmetric about x = 0 so that only the region $0 \le x \le L$ is considered henceforth.

Now, equation (3) becomes

$$m\dot{a}[\partial w/\partial t] = -[Q_p] \qquad (5)$$

across a travelling shear hinge, which using equation (1) can be written

$$m\dot{a}[\partial w/\partial t] = \mp Q_1 [|\gamma|^n]. \qquad (6)$$

Equations (4) and (6) give

$$\dot{a}^2 = \pm c^2 [|\gamma|^n]/[\gamma], \tag{7}$$

where $\dot{a}$ is the velocity of a shear hinge and $c = (Q_1/m)^{1/2}$ is the propagation speed of a shear hinge in a linear work hardening material ($n = 1$). It is evident that the velocity of propagation $\dot{a}$ is not constant when $n \neq 1$ since γ is, generally speaking, a function of x.

If the equation of motion (equation (2) with $P = 0$) in the central region $0 \leq x \leq a$ is integrated, then

$$ma\dot{V} = -Q_o \tag{8}$$

since $Q = 0$ at $x = 0$ and $Q = Q_o$ at $x = a^-$, where a^- refers to the left hand side of the shear hinge at $x = a$ in Figure 2(b). Furthermore, equations (4) and (6) give

$$V = -\dot{a}\gamma(a^+) \tag{9}$$

and

$$ma\dot{V} = -Q_1 |\gamma(a^+)|^n \tag{10}$$

respectively, where a^+ refers to the right hand side of the shear hinge at $x = a$ in Figure 2(b). Equations (9) and (10) may be recast in the form

$$\gamma(a^+) = (m/Q_1)^{\frac{1}{n+1}} V^{\frac{2}{n+1}} \tag{11}$$

and

$$\dot{a} = -(Q_1/m)^{\frac{1}{n+1}} V^{\frac{n-1}{n+1}} \tag{12}$$

which also may be obtained from equations (7) and (11). Now, equation (8) can be written

$$\dot{V} = -\frac{Q_o}{m} \{L + \int_o^t \dot{a} d\bar{t}\}^{-1} \tag{13}$$

or

$$\dot{V} = \frac{-Q_O}{m} \left\{ L - \left(\frac{Q_1}{m} \right)^{\frac{1}{n+1}} \int_O^t V(\bar{t})^{\frac{n-1}{n+1}} d\bar{t} \right\}^{-1} \tag{14}$$

when using equation (12)

If equation (12) is divided by equation (8), then

$$\frac{da}{dV} = \frac{ma}{Q_O} c^{\frac{2}{n+1}} V^{\frac{n-1}{n+1}} \tag{15}$$

which has the solution

$$a = L \exp. \left\{ \frac{(n+1)m}{2nQ_O} c^{\frac{2}{n+1}} \left(V^{\frac{2n}{n+1}} - V_O^{\frac{2n}{n+1}} \right) \right\} \tag{16}^\dagger$$

since $V = V_O$ and $a = L$ when $t = 0$. The final position of the travelling shear hinge (a_f) is given by equation (16) with $V = 0$, i.e.,

$$a_f = L \exp. \left\{ \frac{-(n+1)m}{2nQ_O} c^{\frac{2}{n+1}} V_O^{\frac{2n}{n+1}} \right\} \tag{17}$$

Now, substituting V from equation (16) into equation (11) gives

$$\gamma(a^+) = \left\{ \frac{2nQ_O}{(n+1)Q_1} \log_e \left(\frac{a}{L} \right) + \left(\frac{V_O}{c} \right)^{\frac{2n}{n+1}} \right\}^{\frac{1}{n}} \tag{18}$$

which may also be interpreted as the permanent shear strain since all plastic deformation is confined to the travelling shear hinges. Thus, the permanent transverse displacements in the region $a_f \leq x \leq L$ are $w_f(x) = \int_x^L \gamma(x) \, dx^{\dagger\dagger}$, or

---

† It is evident from equation (16) that the travelling shear hinge never reaches the beam centre.

†† $w_f(x)$ is defined as positive in $-z$ sense.

$$w_f(x) = \int_x^L \left\{ \frac{2nQ_O}{(n+1)Q_1} \, \text{loge} \left(\frac{x}{L} \right) + \left(\frac{V_O}{c} \right)^{\frac{2n}{n+1}} \right\}^{\frac{1}{n}} dx \qquad (19)$$

when using equation (18) with a replaced by x.

In order to express t in terms of V, equation (16) may be substituted into equation (8) and then integrated to give

$$t = \int_V^{V_O} \frac{mL}{Q_O} \, \exp. \left\{ \frac{(n+1)mc}{2nQ_O}^{\frac{2}{n+1}} \left(V^{\frac{2n}{n+1}} - V_O^{\frac{2n}{n+1}} \right) \right\} dV \qquad (20)$$

since $V = V_O$ when $t = 0$. Alternatively, equations (12) and (16) yield the form

$$t = \int_a^L c^{\frac{-2}{n+1}} \left\{ \frac{2nQ_O}{(n+1)mc^{\frac{2}{n+1}}} \, \text{loge} \left(\frac{a}{L} \right) + V_O^{\frac{2n}{n+1}} \right\}^{\frac{1-n}{2n}} da. \qquad (21)$$

The duration of response (t_f) is given by equation (20) with $V = 0$ in the lower limit of the integral, or by equation (21) with $a = a_f$ given by equation (17) in the lower limit of the integral.

The static admissibility of the foregoing kinematically admissible analysis is now examined. The solution of the equilibrium equation (2) in the region of $0 \leq x \leq a$ is $Q = -m\dot{V}x$ or $Q = Q_O x/a$ when using equation (8). This transverse shear force distribution is statically admissible because $Q \leq Q_O$ and incipient plastic flow occurs only at the shear hinge located at $x = a$. Equation (2) in the rigid region $a \leq x \leq L$ reduces to $\partial Q/\partial x = 0$ since $\dot{V} = 0$. Thus, $Q = Q_p(\gamma(a^+))$ which is statically admissible provided $Q_p(\gamma(a^+))$, or $\gamma(a^+)$, decreases with time. However, equation (11) indicates that $\gamma(a^+)$ decreases with time

since V decreases as the shear hinge travels inwards towards the beam centre according to equation (16). It is apparent that the foregoing theoretical predictions are both kinematically and statically admissible and therefore exact within the spirit of the theory.

The transverse shear force in the outer zone $a \leq x \leq L$ is $Q = Q_p(\gamma(a^+))$ and therefore the associated bending moment distribution according to $M = \int Q \, dx$ is $M = M^* - Q_p(\gamma(a^+))(L-x)$, where M^* is the magnitude of the bending moment at the supports. Similarly, $Q = Q_o x/a$ and $M = M^h + Q_o(x^2-a^2)/2a$ in the inner region $o \leq x \leq a$, where M^h is the value of the bending moment at the travelling plastic shear hinge. However, the bending moment must be continuous across a travelling shear hinge when rotatory inertia is neglected [11]. Thus, $M^h = M^* - Q_p(\gamma(a^+))(L-a)$.

Now, the bending moment (M) does not enter into the yield condition because an ideal fibre-reinforced beam is inextensible along its length and therefore any value of M required for equilibrium may be supported without any associated strains and without penetrating the yield surface. It is evident, therefore, that the foregoing theoretical solution is valid for any value of M^* at the supports. In particular, the theoretical predictions are valid for "simple" supports with $M^* = 0$ as well as the fully clamped case.

Impulsive Loading of a Clamped Beam with Linear Strain Hardening (n = 1)

The theoretical analysis in the previous section is particularly simple when $n = 1$ since the shear hinge then propagates with a

constant velocity $\dot{a} = -c$ according to equation (12).

Integration of equation (13) with $n = 1$ and $V = V_o$ when $t = 0$ gives

$$V = V_o + \frac{\Omega_o}{mc} \log_e (1 - \frac{ct}{L}) \tag{22}$$

which predicts a duration of response

$$t_f = \frac{L}{c} \left[1 - \exp. \{ \frac{-mcV_o}{Q_o} \} \right] \tag{23}$$

when $V = 0$. Thus, the final position of the shear hinge is $a_f = L - ct_f$, or

$$a_f = L \exp. \{ \frac{-mc V_o}{Q_o} \} . \tag{24}$$

The permanently deformed profile of the beam is $w_f = \int_x^L \gamma_f(x) dx^\dagger$ which when using $\gamma_f(x) = \gamma(a^+)$, equation (11) with $n = 1$ and equation (22) gives

$$w_f = \frac{V_o L}{c} (1 - \frac{x}{L}) - \frac{Q_o L}{Q_1} \{ 1 + \frac{x}{L} \log_e(\frac{x}{L}) - \frac{x}{L} \} \tag{25}$$

for $a_f \le x \le L$ and

$$w_f = L \left[\frac{V_o}{c} + \frac{Q_o}{Q_1} \{ \exp. (\frac{-mcV_o}{Q_o}) - 1 \} \right] \tag{26}^\dagger$$

in the region $0 \le x \le a_f$.

Introducing the non-dimensional parameters

$$q = Q_1/Q_o \text{ and } \lambda = V_o/c \tag{27a, b}$$

allows equations (23), (24) and (26) to be recast in the form

$$ct_f/L = 1 - e^{-q\lambda} \tag{28}$$

$\dagger$ w_f is defined positive in $-z$ sense.

$$a_f/L = e^{-q\lambda} \tag{29}$$

and
$$w_f/L = \lambda + (e^{-q\lambda} - 1)/q \tag{30}$$

respectively, while equation (11) or equation (18) indicates

$$\gamma_f(max) = \lambda \tag{31}$$

at $x = L$.

Discussion

It is evident from equation (29) that the final position of a shear hinge (a_f/L) is near the supports when $\beta = \lambda q$ is small and $n = 1$. In this circumstance, a large central portion of a beam remains undeformed and the plastic deformation is confined to the vicinity of the supports as shown in Figure 3. However, the size of the deformed zone near the supports of a beam increases as the strain-hardening index n is decreased, while the associated permanent transverse deflections decrease according to equation (19) and as indicated in Figure 3. The influence of the material strain hardening parameter (q) and strain hardening index (n) on the maximum permanent transverse displacements according to equation (19) is shown in Figure 4, where the abscissa is $\lambda(q)^{1/2} = V_o(m/Q_o)^{1/2}$.

The permanent transverse deflection at the mid-span of a fully clamped rigid perfectly plastic isotropic beam loaded impulsively with a uniform velocity V_o is [12]

$$w_{fi} = mV_o^2 L^2/(6M_o) \tag{32}$$

where $M_o = \sigma_o H^2/4$ and H is the thickness of a beam with a rectangular

cross-section. If plastic yielding of a cross-section is controlled by the Tresca yield condition, then the fully plastic transverse shear force is $Q_o = \sigma_o H/2$ and therefore $M_o = Q_o H/2$. Thus, equation (32) may be written

$$w_{fi}/L = \lambda^2 q L/(3H) \tag{33}$$

which when divided by equation (30) for $n = 1$ gives

$$\frac{w_{fi}}{w_f} = \frac{\beta^2}{3(\beta - 1 + e^{-\beta})} \left(\frac{L}{H}\right) \tag{34}$$

where $\beta = \lambda q$. Equation (34) predicts

$$w_{fi}/w_f = 2L/(3H) \tag{35}$$

when $\beta \ll 1$ (e.g., very small strain hardening parameter Q_1). the variation of w_{fi}/w_f with L/H and β according to equations (34) and (35) is shown in Figure 5. These results indicate that the permanent transverse displacements of this ideal fibre-reinforced (strongly anisotropic) rigid plastic beam with $n = 1$ are much smaller than those of an "equivalent" rigid perfectly plastic isotropic beam, even when $Q_1 \rightarrow o$ (i.e., $\beta \rightarrow o$).

The response duration of a fully clamped rigid perfectly plastic isotropic beam is

$$t_{fi} = m V_o L^2/(4M_o), \tag{36}$$

while equation (23) or equation (28) predict

$$t_f = m V_o L/Q_o \tag{37}$$

when $\beta \ll 1$ (e.g., very small strain hardening parameter Q_1). Thus,

$$t_{fi}/t_f = L/(2H) \tag{38}$$

which indicates that the response duration of an ideal fibre-reinforced (strongly anisotropic) rigid-plastic beam is considerably less than an "equivalent" rigid perfectly plastic isotropic beam.

Conclusions

This article examines the dynamic plastic response of an impulsively loaded ideal fibre-reinforced (strongly anisotropic) rigid-plastic beam with material strain hardening. It is demonstrated that the response duration and permanent transverse displacements are significantly less than the corresponding quantities in an "equivalent" rigid perfectly plastic isotropic beam. Thus, it appears that a potential for considerable weight savings may exist for energy absorbing systems made from materials characterised as ideal fibre-reinforced rigid-plastic. However, experimental investigations are required to establish if the ideal fibre-reinforced material model is useful for strongly anisotropic beams loaded dynamically. The particular case examined herein offers an attractive experimental arrangement for experimental work since similar rigid-plastic beams have been investigated by many authors.

References

1.   Spencer, A.J.M., "Dynamics of Ideal Fibre-Reinforced Rigid-Plastic Beams", J. Mechs. Phys. Solids, 22, 147-159, 1974.

2.   Jones, N., "Dynamic Behavior of Ideal Fibre-Reinforced Rigid-Plastic Beams", J. App. Mechs., 43, 319-324, 1976.

3.   Spencer, A.J.M., "A Note on an Ideal Fibre-Reinforced Rigid-Plastic Beam Brought to Rest by Transverse Impact", Mechs. Res. Comm., 3, 55-58, 1976.

4.   Shaw, L. and Spencer, A.J.M., "Impulsive Loading of Ideal Fibre-Reinforced Rigid-Plastic Beams. I. Free Beam Under Central Impact", Univ. of Nottingham Rep., 1976.

5.   Shaw, L. and Spencer, A.J.M., "Impulsive Loading of Ideal Fibre-Reinforced Rigid-Plastic Beams. II. Beam with End Supports", Univ. of Nottingham Rep., 1976.

6.   Shaw, L. and Spencer, A.J.M., "Impulsive Loading of Ideal Fibre-Reinforced Rigid-Plastic Beams. III Cantilever Beam", Univ of Nottingham Rep. 1976.

7.   Symonds, P.S., and Jones, N. "Impulsive Loading of Fully Clamped Beams with Finite Plastic Deflections and Strain-Rate Sensitivity", Int. J. Mech. Sci., 14, 49-69, 1972.

8.   Symonds, P.S., "Survey of Methods of Analysis for Plastic Deformation of Structures under Dynamic Loading", Brown Univ. Rep. BU/NSRDC/1-67, 1967.

9.   Johnson, W., "Impact Strength of Materials", Crane Russak, New York, E. Arnold, London, 1972.

10. Jones, N., "A Literature Review of the Dynamic Plastic Response of Structures", The Shock and Vibration Digest, Vol. 7, No. 8, 89-105, 1975.

11. Symonds, P.S., "Plastic Shear Deformations in Dynamic Load Problems", Engineering Plasticity, Ed. J. Heyman and F.A. Leckie, 647-664, 1968.

12. Martin, J.B., and Symonds, P.S., "Mode Approximations For Impulsively-Loaded Rigid-Plastic Structures", Proc. A.S.C.E., 92, No. EM5 43-66, 1966.

-14-

Titles of Figures

Figure 1.   Sign Convention.

Figure 2.   (a) Impulsive Loading of a Fully Clamped Beam

(b) Transverse Displacement Profile at time t.

Figure 3.   Permanent Transverse Displacement Profiles according

to Equation (19) with $\lambda = 0.1$ and $q = 4$.

①: $n = 1$, $a_f/L = 0.6703$, $ct_f/L = 0.3297$, $w_{max}/L = 0.01758$

②: $n = 1/3$, $a_f/L = 0.07967$, $ct_f/L = 0.00635$, $w_{max}/L = 0.01259$

③: $n = 1/5$, $a_f/L = 0.00381$, $ct_f/L = 0.00152$, $w_{max}/L = 0.01055$

Figure 4.   Influence of Material Strain Hardening Parameter

$(q = Q_1/Q_o)$ and strain hardening index n on the

Maximum Permanent Transverse Displacements of Impulsively

Loaded Beams According to Equation (19).

Figure 5.   Comparison of Maximum Permanent Transverse Displacements

of Rigid Perfectly Plastic Isotropic (w_{fi}) and Ideal

Fibre-Reinforced (Strongly Anisotropic) Rigid Plastic

(w_f) Beams Loaded Impulsively According to Equations

(34) and (35).
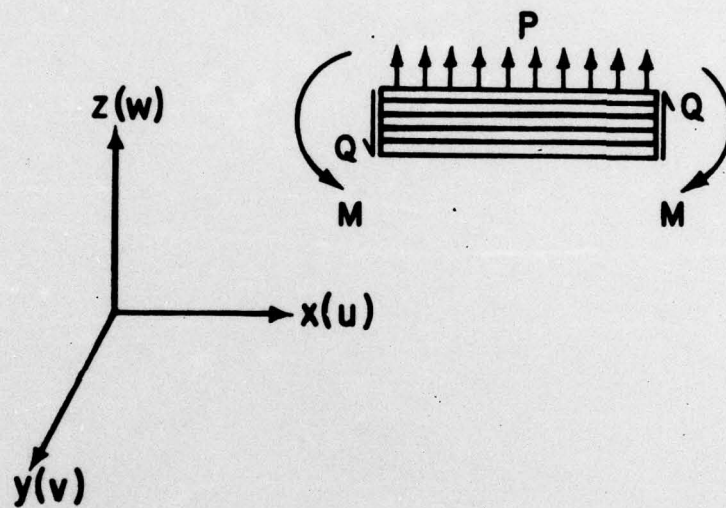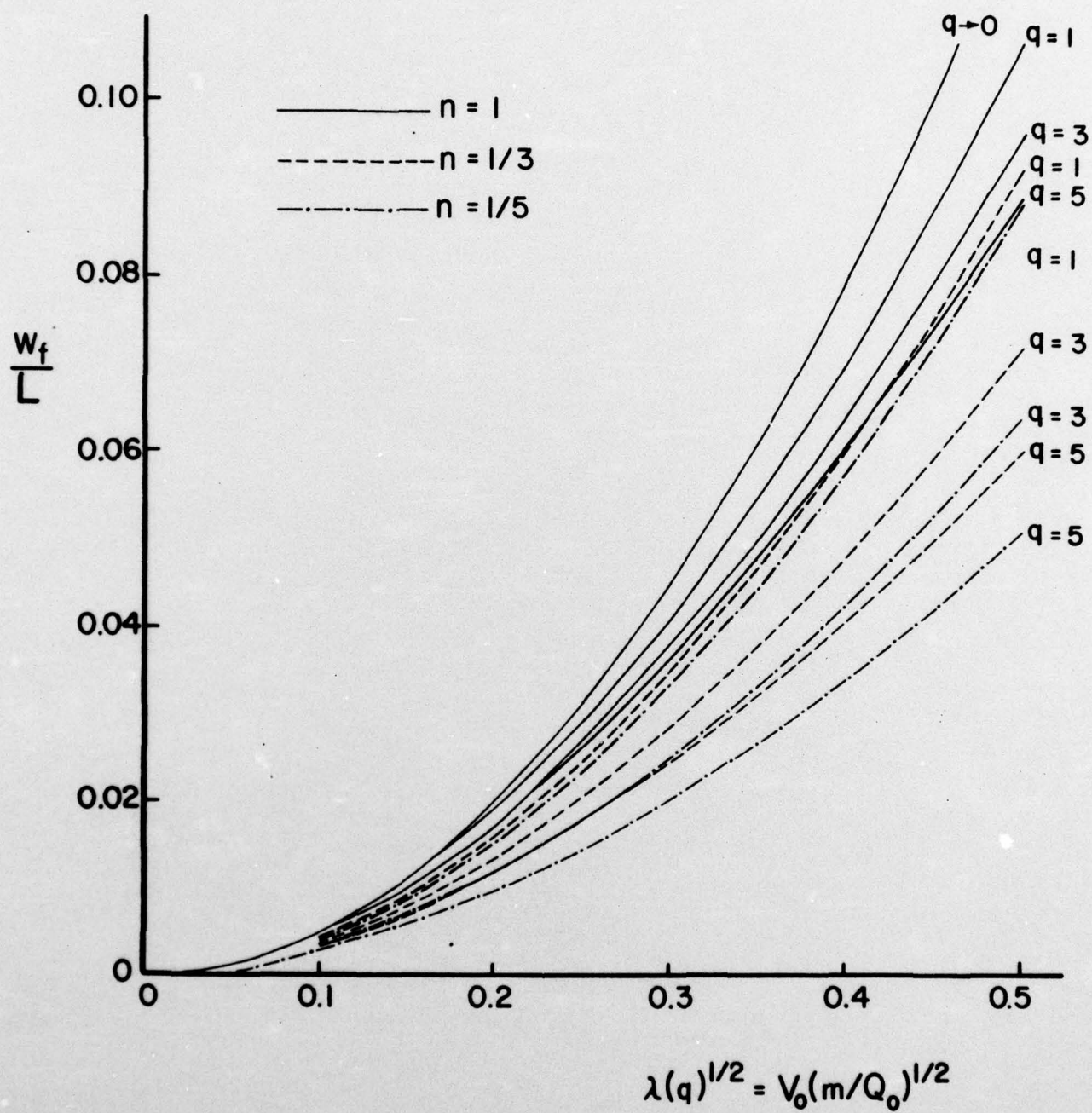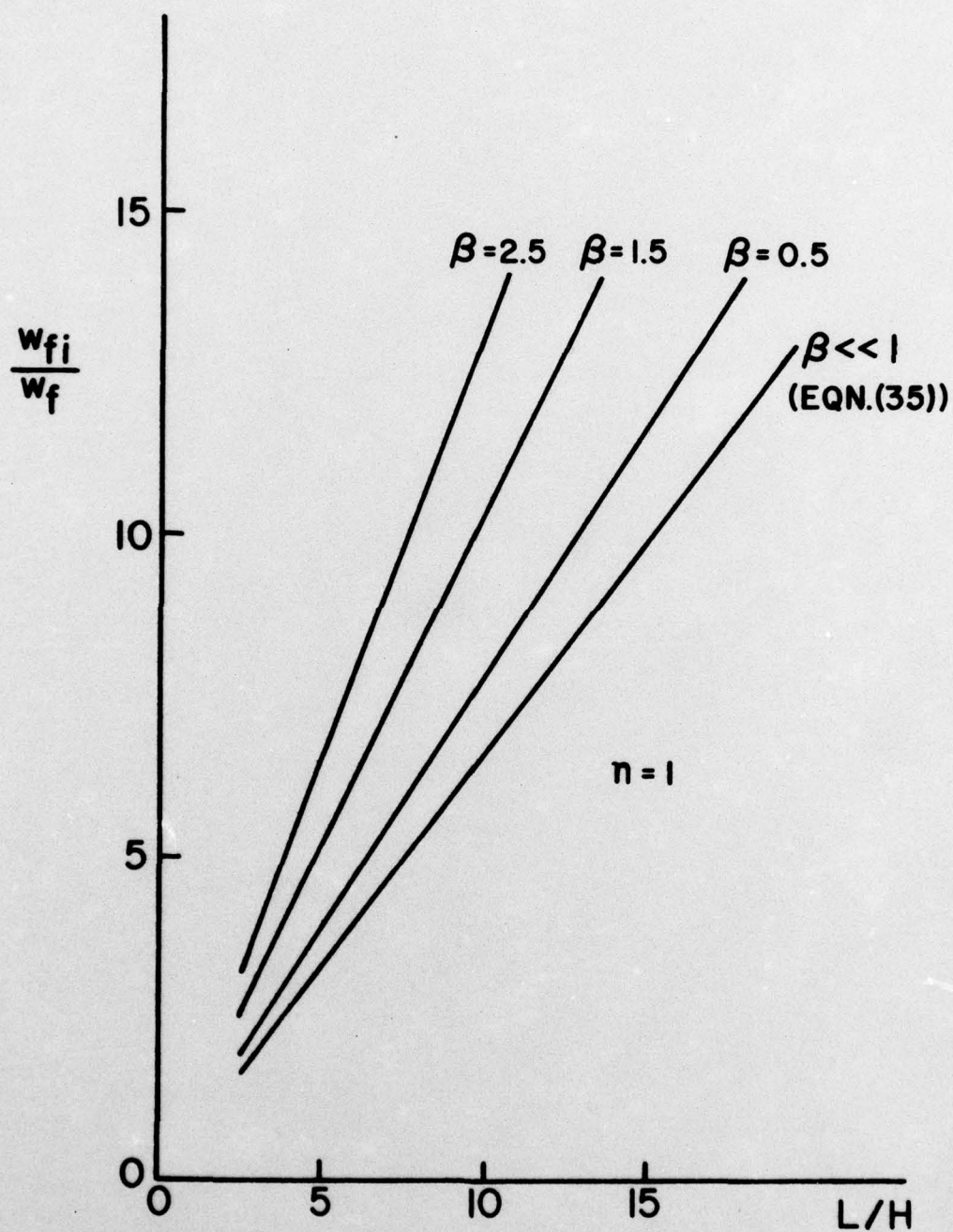
FIGURE 1
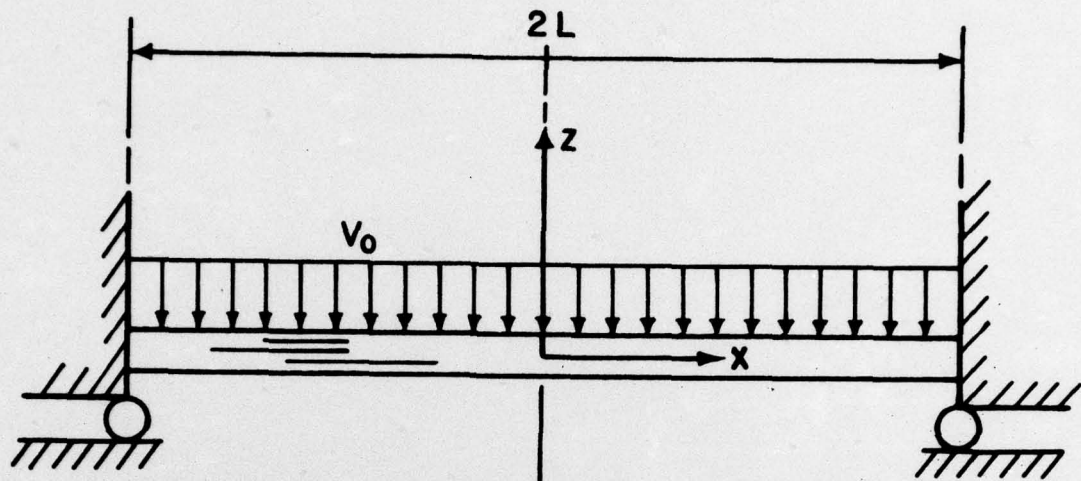
(a)
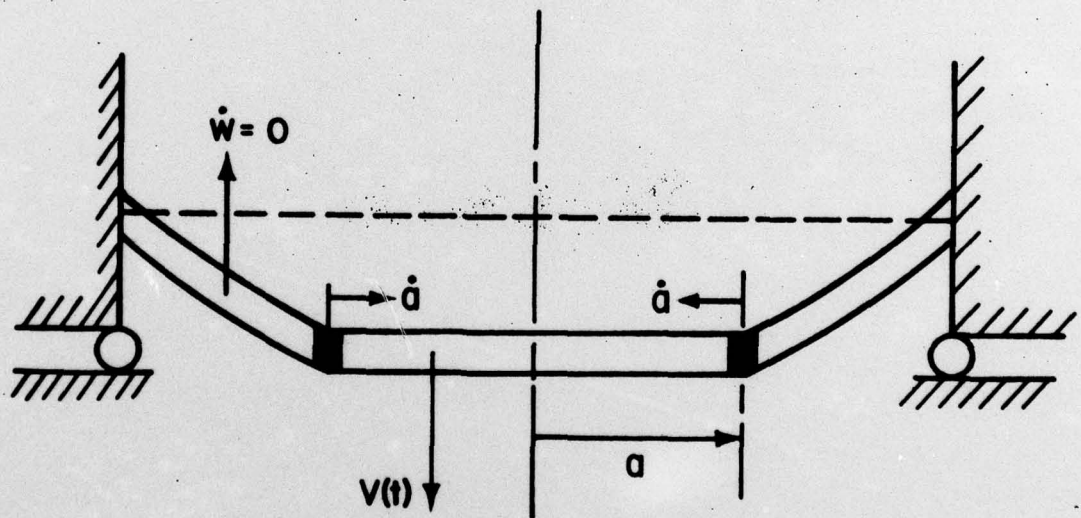
(b)
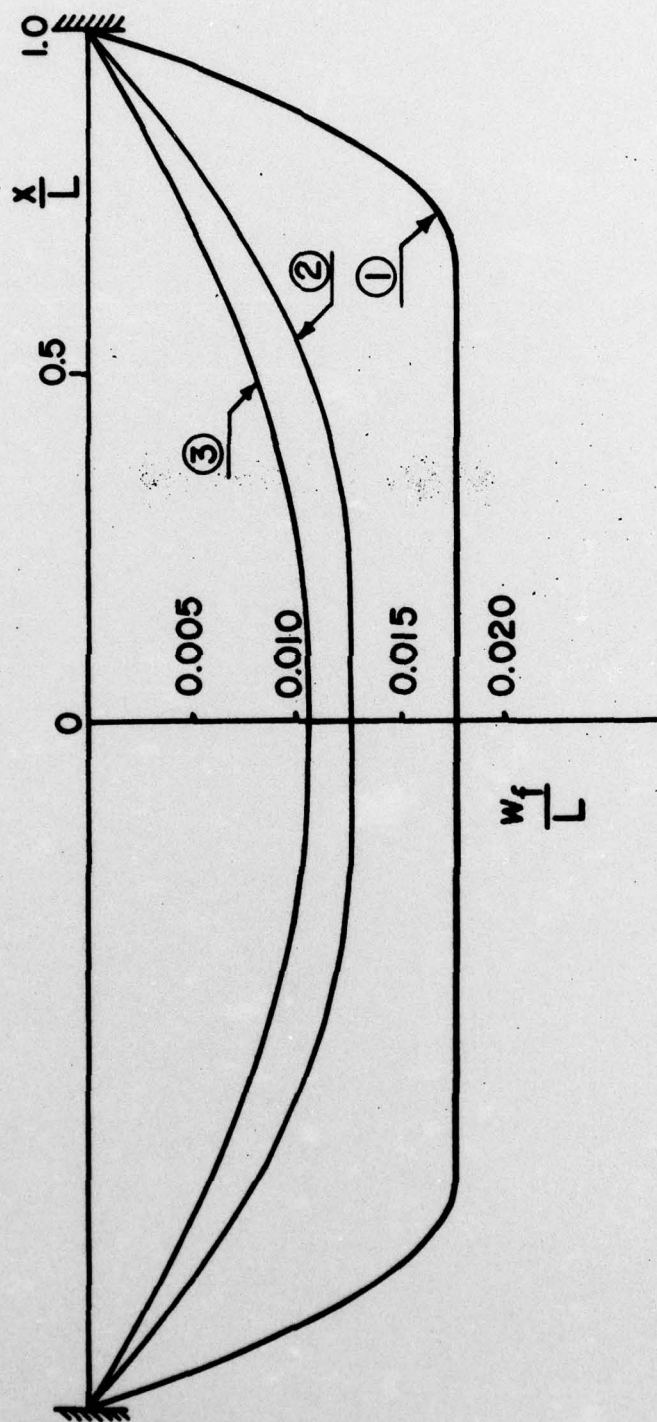
FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

M.I.T. Department of Ocean Engineering
Report No. 77-3

IMPULSIVE LOADING OF AN IDEAL FIBRE-REINFORCED
RIGID-PLASTIC BEAM, by F. Laudiero and Norman Jones
March 1977, 19 pgs., illustrated. UNCLASSIFIED

This article examines the dynamic plastic response of an impulsively loaded ideal fibre-reinforced (strongly anisotropic) rigid-plastic beam. It is demonstrated that the response duration and permanent transverse displacements are significantly less than the corresponding quantities in an "equivalent" rigid perfectly plastic isotropic beam.

Impulsive Loading

Plastic

Fibre-Reinforced

Beams

M.I.T. Department of Ocean Engineering
Report No. 77-3

IMPULSIVE LOADING OF AN IDEAL FIBRE-REINFORCED
RIGID-PLASTIC BEAM, by F. Laudiero and Norman Jones
March 1977, 19 pgs., illustrated. UNCLASSIFIED

This article examines the dynamic plastic response of an impulsively loaded ideal fibre-reinforced (strongly anisotropic) rigid-plastic beam. It is demonstrated that the response duration and permanent transverse displacements are significantly less than the corresponding quantities in an "equivalent" rigid perfectly plastic isotropic beam.

Impulsive Loading

Plastic

Fibre-Reinforced

Beams

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 77-3	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Impulsive Loading of an Ideal Fibre-Reinforced Rigid-Plastic Beam.		5. TYPE OF REPORT & PERIOD COVERED Interim rept.
		6. PERFORMING ORG. REPORT NUMBER 77-3
7. AUTHOR(s) F. Laudiero Norman Jones		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0195
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Ocean Engineering Massachusetts Institute of Technology Cambridge, Massachusetts 02139		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Structural Mechanics Program O.N.R. Arlington, Virginia 22217		12. REPORT DATE March 1977
		13. NUMBER OF PAGES 26p.
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited Distribution

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Impulsive loading
Plastic
Fibre-Reinforced
Beams

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This article examines the dynamic plastic response of an impulsively loaded ideal fibre-reinforced (strongly anisotropic) rigid-plastic beam. It is demonstrated that the response duration and permanent transverse displacements are significantly less than the corresponding quantities in an "equivalent" rigid perfectly plastic isotropic beam.

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE
1 JAN 73
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

406 856